Symmetry in Regular Polygons

A figure has reflection symmetry if you can reflect the figure over a line so that the image will coincide with the original figure. The line you reflect over is called a line of symmetry or a mirror line. A figure has rotational symmetry if you can rotate it some number of degrees about some point so that the rotated image will coincide with the original figure. In this exploration, you’ll look for reflection and rotation symmetries of regular polygons.

Sketch and Investigate

1. Construct a regular polygon and its interior. You can use an equilateral triangle, a square, a regular pentagon, or a regular hexagon. You may want to have different groups in your class investigate different shapes.

2. Construct a line.

3. Mark the line as a mirror and reflect the polygon interior over it.

4. Give the image a different color.

5. Drag the line until the image of your polygon coincides exactly with the original.

Q1 When a reflection image coincides with the original figure, the reflection line is a line of symmetry. Describe how the line of symmetry is positioned relative to the figure.

6. Drag the line so that it is a different line of symmetry. Repeat until you have found all the reflection symmetries of your polygon.

Q2 Fill in one entry in the table below: the number of reflection symmetries for your polygon. (Note: Be careful not to count the same line twice!) You’ll come back to fill in other entries as you gather more information.

<table>
<thead>
<tr>
<th>Number of sides of regular polygon</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>...</th>
<th>n</th>
</tr>
</thead>
<tbody>
<tr>
<td>Number of reflection symmetries</td>
<td></td>
<td></td>
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<td></td>
<td></td>
</tr>
<tr>
<td>Number of rotation symmetries</td>
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</tbody>
</table>
Symmetry in Regular Polygons (continued)

Next, you’ll look for rotation symmetries.

7. Move the line so that the reflected image is out of the way.

8. If the polygon’s center doesn’t already exist, construct it.

9. Use the Segment tool to construct an angle.

10. Measure the angle.

11. In Preferences, set Angle Units to directed degrees.

12. Mark the center of the polygon as a center for rotation and mark the angle measurement. Rotate the polygon interior by this marked angle measurement.

13. Give the rotated image a different color.

14. Change the angle so that the rotated image fits exactly over the original figure.

Q3 What angle measure causes the figures to coincide?

Polygon: _______________ Rotation angle: _______________

15. Continue changing your angle to find all possible rotation symmetries of your polygon.

Q4 Count the number of times the rotated image coincides with the original when rotating from 0° to 180° and from –180° back to 0°. In your chart on the preceding page, record the total number of rotation symmetries you found. (Note: Count no revolutions, or 0°, as one of your rotation symmetries.)

Q5 Combine the results from other members of your class to complete your chart with the reflection and rotation symmetries of other regular polygons.

Q6 Use your findings to write a conjecture about the reflection and rotation symmetries of a regular n-gon. Include in your conjecture a statement about the smallest angle of rotational symmetry greater than 0.